Performance Based Learning and Assessment Task

Shuffle or Nano?

I. ASSESSSMENT TASK OVERVIEW & PURPOSE:

This task is to provide students with the opportunity to analyze a real world situation and create a mathematical model. Students will restrict the domain to determine models for certain segments of real world data. Students will create functions and determine if they are appropriate for making predictions.

II. UNIT AUTHOR:

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III. COURSE:

AFDA

IV. CONTENT STRAND:

Functions

V. OBJECTIVES:

- Create scatterplots of real world data.
- Determine which function best serves as a model for the entire data set
- Realize that more than one model may be necessary to best describe real world data.
- By restricting the domain, the student will find appropriate models for certain portions of the data set.
- Determine if a model is an appropriate predictor of future data.

VI. REFERENCE/RESOURCE MATERIALS:

Copy of Performance Based Assessment including real world data.
Copy of Rubric
Graphing Calculator
Optional: Graph Paper
Optional: Computer with spreadsheet software

VII. PRIMARY ASSESSMENT STRATEGIES:

The students will utilize the attached assessment list/scoring rubric to determine if he has met the criteria for the task.
The teacher will utilize the attached assessment/scoring rubric to determine the scoring of each task.
Accommodations: Students may graph the data using graph paper or technology such as a graphing calculator or spreadsheet program. Analysis may be completed using a graphing calculator or a spreadsheet program.

VIII. EVALUATION CRITERIA:
Students will be evaluated using the attached assessment list/scoring rubric. A benchmark is attached.

IX. INSTRUCTIONAL TIME:
60 – 90 minutes

Activity/Task 1 Title

Strand
Functions

Mathematical Objective(s)
- Create scatterplots of real world data.
- Determine which function best serves as a model for the entire data set
- Realize that more than one model may be necessary to best describe real world data.
- By restricting the domain, the student will find appropriate models for certain portions of the data set.
- Determine if a model is an appropriate predictor of future data.

Related SOL

AFDA.1 (functions: domain and range, increasing/decreasing)
AFDA.3 (mathematical modeling: curve of best fit, predictions)

NCTM Standards List all applicable NCTM standards related to each task/activity. Example:

- Use symbolic algebra to represent and explain mathematical relationships.
- Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships.
- Draw reasonable conclusions about a situation being modeled.
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others.

Additional Objectives for Student Learning (include if relevant; may not be math-related):

NA

Materials/Resources

Copy of Performance Based Assessment
Copy of Rubric

Graphing Calculator
Graph Paper
Optional: Computer with spreadsheet software

Assumption of Prior Knowledge

- Students' prior knowledge should include characteristics of linear, exponential, polynomial, and logarithmic functions and whether a variable is discrete or continuous.
- The students should be operating at the Analysis level on the Van Hiele scale with respect to function analysis.
- Students should be able to identify portions of graphs of real world data that have characteristics of linear, exponential, polynomial, or logarithmic functions.
- Identify/ predict what students may find difficult or confusing or have misconceptions about?
- The student should have previously investigated and analyzed functions both algebraicly and graphically. In particular, students should have experience in identifying the domain and range of a function, including limited and discontinuous domains. Students should be able to determine if a variable is discrete or continuous and how that impacts a graph of the data. Additionally, students should have had practice at determining if predictions based on interpolated and extrapolated data is accurate.
- Common difficulties relate to determining if a variable is discrete or continuous and how that impacts the domain and/or range of a model and the appearance of a graph containing that variable.
- The context for this task is sales of iPods over time. It could be applied to the sales of other items.

Introduction: Setting Up the Mathematical Task

This task is intended to be used as an assessment of the knowledge level of an individual student or pair of students.

- The goal of this task is to find a model that illustrates the Total Annual Sales of iPods over time.
- This task should be completed in a 90 minute class period.
- Begin by handing out the Performance Based Assessment with the data.
- Ask students to read over the task. Provide time for student to ask clarifying questions.
- Explain that students are to complete the task in writing. The tools they may use include the graphing calculator, graph paper, and a computer with spreadsheet software (optional) and a printer to print out necessary information (optional).

Student Exploration

This task is intended to be used as an assessment of the learning that occurs from the task for an individual or pair of students.

- The students should be working individually or as a pair.
- In addition to the task and assessment list, the teacher should make the following items available to the students: graph paper, graphing calculator, computer with spreadsheet software (optional), printer (optional).
- Since this is an assessment, the teacher should be monitoring the students, and, if appropriate for the group of students, asking questions of the individual or group to help get the student(s) thinking in the right direction.

Monitoring Student Responses

Students are expected to communicate their responses in writing.

- Graphs and equations should be included in the responses.
- Appropriate statistical analysis should be included in the response. This includes finding reasonable models based on the scatterplot and selecting the best model based on the r value for linear models or R^2 value for non-linear models.

Assessment List and Benchmarks

- The students will utilize the attached assessment list/scoring rubric to determine if he has met the criteria for the task.
- The teacher will utilize the attached assessment/scoring rubric to determine the scoring of each task.
- A benchmark is provided at the end of the activity.

Nano or Shuffle?

The attached table lists the number of iPods sold worldwide per fiscal quarter from the time it was released to the present.

1. Describe the data in the chart.

2. Graph the data; describe what the graph of the data tells you.

3. Identify a model for the data; discuss why you think this model is most appropriate for this data.

4. When you look at the graph, note the trends in various sections of the graph. Could different models be used to describe the data in these sections? Why or why not?

5. Identify the model that would provide the best predictions for the data. Why is this model the best?

6. Apple's fiscal year ends in September. Fiscal Q1 is Oct–Dec of previous year; this means Q1 of 2008 is Oct–Dec of 2007, Q2 of 2008 is Jan–Mar of 2008 and so on.
   Which quarter is the best predictor of the total number of iPods sold for the year? Support your answer using appropriate mathematics.

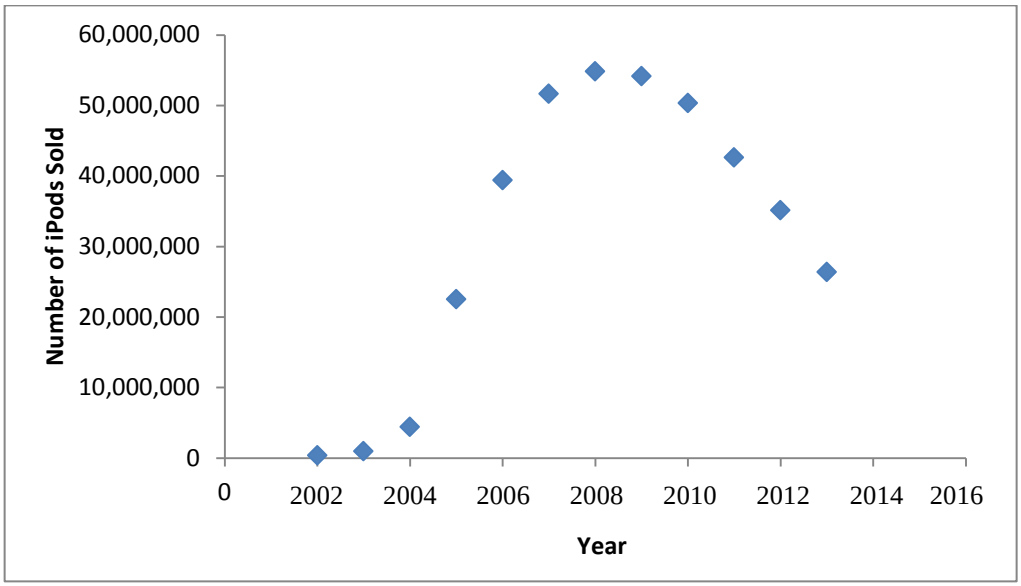
Ipod Sales per Quarter

Fiscal Year	Q1	Q2	Q3	Q4	Total
2002	125,000	57,000	54,000	140,000	376,000
2003	219,000	78,000	304,000	336,000	937,000
2004	733,000	807,000	860,000	2,016,000	4,416,000
2005	4,580,000	5,311,000	6,155,000	6,451,000	22,497,000
2006	14,043,000	8,526,000	8,111,000	8,729,000	39,409,000
2007	21,066,000	10,549,000	9,815,000	10,200,000	51,630,000
2008	22,121,000	10,644,000	11,011,000	11,052,000	54,828,000
2009	22,727,000	11,013,000	10,215,000	10,177,000	54,132,000
2010	20,970,000	10,885,000	9,410,000	9,050,000	50,315,000
2011	19,446,000	9,020,000	7,540,000	6,622,000	42,628,000
2012	15,397,000	7,673,000	6,751,000	5,344,000	35,111,000
2013	12,679,000	5,633,000	4,569,000	3,498,000	26,379,000
2014	6,049,000	2,761,000	2,926,000		

http://en.wikipedia.org/wiki/File:Ipod_sales_per_quarter.svg

Benchmark

1. The data in the chart describes the number of iPods sold worldwide per fiscal quarter from the time it was released to the present. The data shows that each year the largest amount of sales occurred during Quarter 1. The total sales shows relatively low sales the first two years as compared with the other years. Sales increased greatly each year from 2004 from about 2008, but then sales began to decrease.



2. Sales were relatively low the first two years, then appeared to grow exponentially until 2008. From 2008 through 2013, sales decreased in an approximately linear fashion.

   Prior to 2002, people relied on other portable devices such as the Walkman to bring their music with them. When MP3 players like the iPod first came out, people were slow to make the transition. However, they rapidly gained in popularity. As more and more people began using mobile phones, instead of putting their music on an iPod or other MP3 device, people put their music on their phones so they would only have to carry one device.

3. a. Because of the curves in the data display, I explored the quadratic, cubic, and quartic functions as possible models for the data. Although the all three models follow the curve of the data points somewhat, the quartic model has the closest fit.

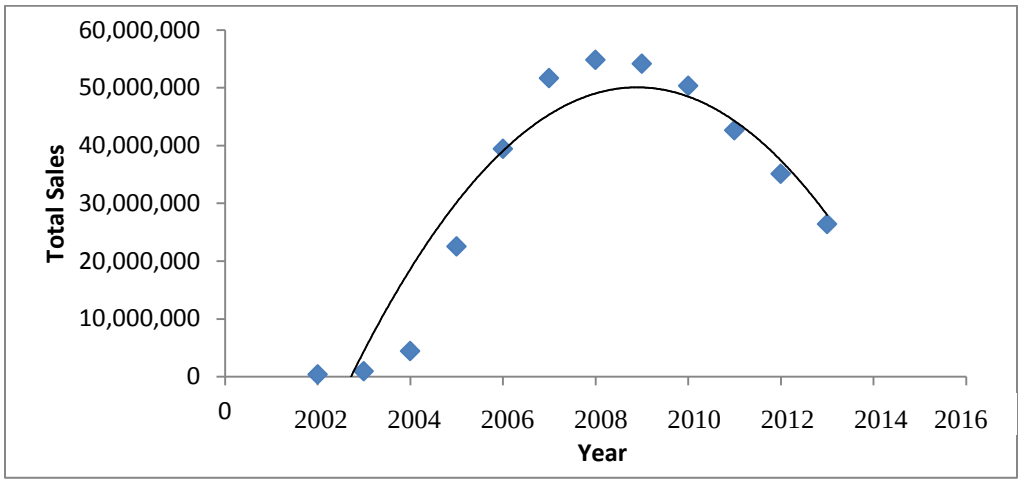
   Quartic Model: $y = 55140.1x^4 - 442910380.3x^3 + (1.3341182 \text{ X } 10^{12})x^2 - (1.786032 \text{ X } 10^{15})x + (8.9663241 \text{ X } 10^{17})$

   b. The quartic model is the best for the given data, as it fits the data the best and has the highest R^2 value.

   c. The quartic model fits the given data quite well and could be used for interpolation within the data, it would not be good for extrapolation even in the near future, as the

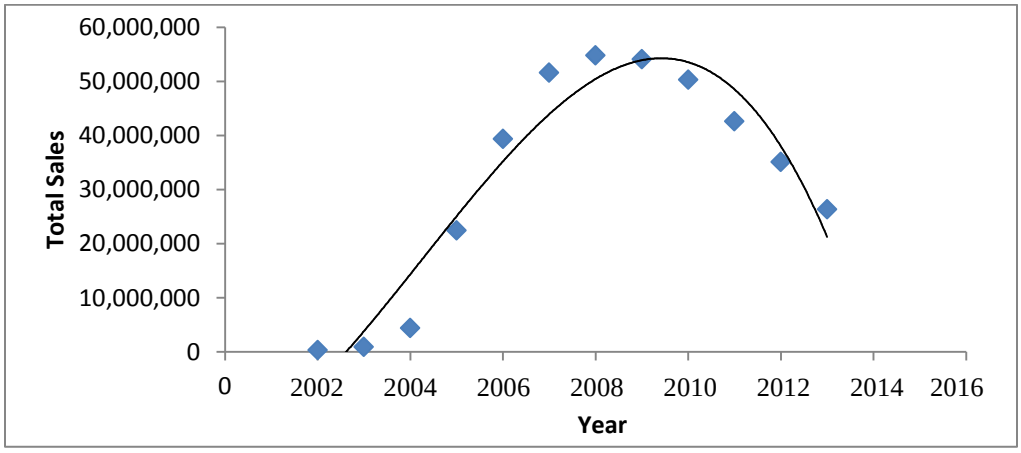
graph turns upward after 2013. We can see from the first two quarters of 2014 that sales are still continuing to decrease.

Quadratic Model: $y = -1312875.6x^2 + 5274858359x - 5.29826801 \text{ X } 10^{12}$; $R^2 = .89$



Cubic Model: $y = -135256.2x^3 + 813267295.3x^2 - (162992 \text{ X } 10^{12})x + 1.0889639 \text{ X } 10^{15}$;

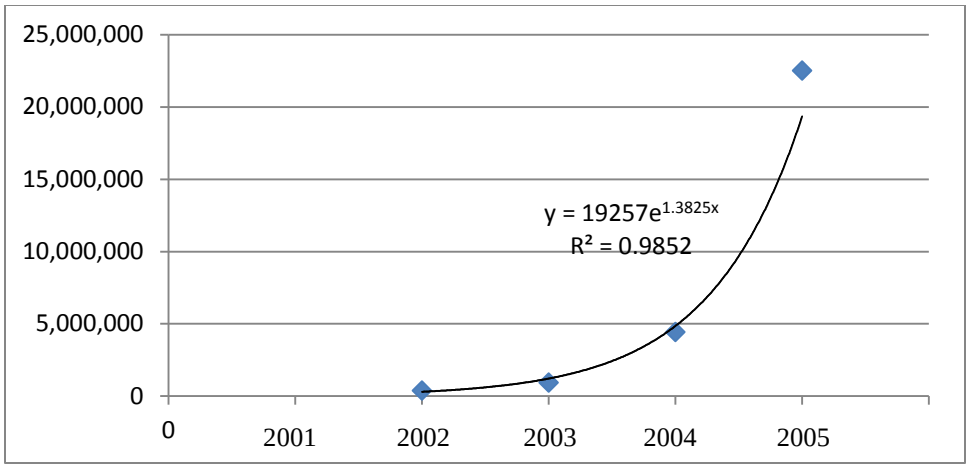
$R^2 = .93$



Quartic Model: $y = 55140.1x^4 - 442910380.3x^3 + (1.3341182 \text{ X } 10^{12})x^2 - (1.786032 \text{ X } 10^{15})x + (8.9663241 \text{ X } 10^{17})$; $R^2 = .99$

5. The 4 data points from 2002 to 2005 yields the following graph. These points demonstrate exponential growth during this time period:

$y = 19257e^{1.3825x}$; $R^2 = .985$ Domain: $2002 \leq x \leq 2005$



This model for the given domain fits the points very well, with an R^2 value of 0.985.

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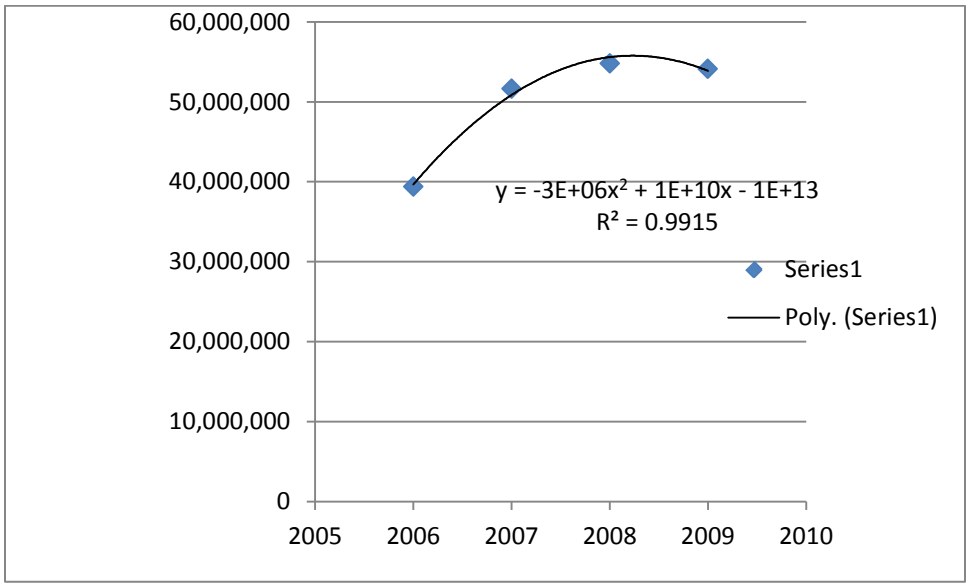
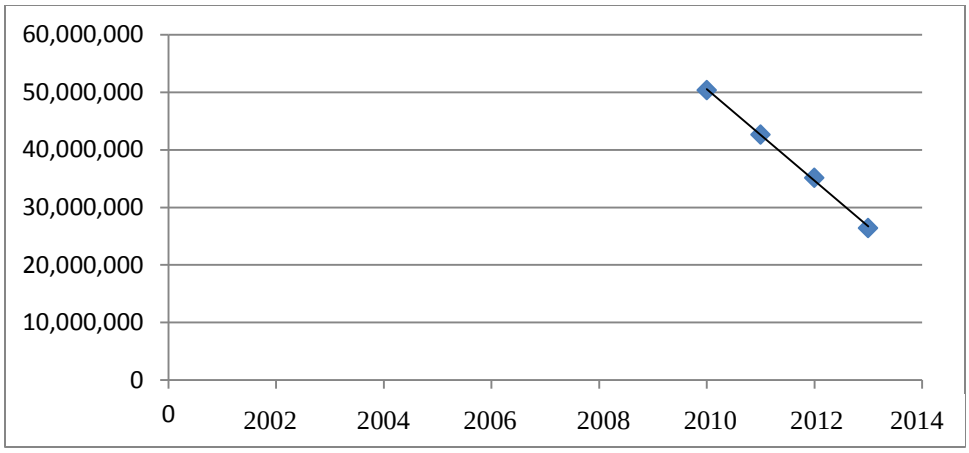
The four data points from 2006 yield the following graph. These points fit a quadratic equation.
$y = -2229250x^2 + (1.2970175 \text{ X } 10^{10}) - 1.302351 \text{ X } 10^{13}$, $R^2 = .992$
Domain $2006 \leq x \leq 2009$



The four data points from 2010 to 2013 yield the following graph. These points demonstrate a linear decrease during this time period.

$y = -7932500x + (1.5994832 \text{ X } 10^{10})$; r = -.999, R2 = .999
Domain: $2010 \leq x \leq 2013$



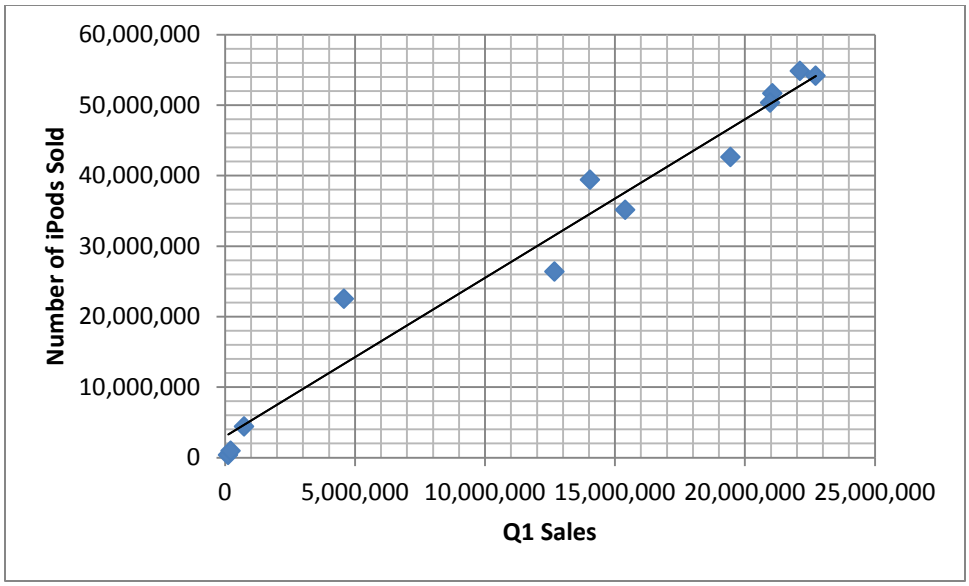
The linear model fits the data in the domain very well, with r = -0.999, indicating a very strong linear correlation, and $R^2 = 0.999$.

The best model would be a piecewise function that would incorporate all of these functions:

For $2002 \leq x \leq 2005$ $y = 19257e^{1.3825x}$
For $2006 < x < 2009$ $y = -2229250x^2 + (1.2970175 \text{ X } 10^{10}) – 1.302351 \text{ X } 10^{13}$
For $2010 \leq x \leq 2013$ $y = -7932500x + (1.5994832 \text{ X } 10^{10})$;

It is the best because it best fits the data in each section better than any single model. Each section of this model would be very good for prediction within the given part of the domain because of the high R^2 values, but not for extrapolation since the sales model changes considerably every few years. The last piece from 2010 to 2013 would be acceptable for extrapolation in 2014 and maybe 2015, but would not be recommended for years further outside its domain.

6. The graph of Q1 sales vs Total Annual Sales is approximately linear.
$y = 2.249x + 3004152.9$; $r = .981$; $R^2 = .96$. Because r and R^2 are both high, Q1 sales would be a good predictor for iPod sales for the year.



The regression equation y = 2.249x + 3004152.9 has an r-value of 0.981, indicating a strong linear relationship between Q1 sales and the number of iPods sold that year, therefore it would be reasonable to use Q1 sales to predict the number of iPods sold.

Scoring Rubric

Item	0	1	2	3
1	Student did not describe the data in the chart	Student made one accurate statement about the data in the chart.	Student made two accurate statements about the data in the chart.	Student made three or more accurate statements about the data in the chart.
2	Student did not make a graph	Student made a graph but did not label axis or scale correctly.	Student made a graph. He labeled the axes or had the scales correct.	Student made a graph. He labeled the axis correctly and had the scales correct.
3	The student did not find a model for the data.	The student suggested a model but did not support why it was the most appropriate model.	The student explored at least two models, determined one was more appropriate, and correctly discussed why his choice was the most appropriate model.	The student explored at least two models, determined one was more appropriate, correctly discussed why his choice was the most appropriate model, and discussed its accuracy.
4	The student did not identify trends in various sections of the graph.	The student identified trends that appeared in at least two sections of the graph but did not suggest or incorrectly suggested models that would fit both of these trends.	The student identified trends that appeared in at least two sections of the graph. He correctly suggested a model for one of the trends.	The student identified at least two trends in various sections of the graphs and correctly suggested a model that would fit each of these trends.
5	The student did not find a model or found an inappropriate model.	The student found a model and either indicated the domain for the model or discussed why it was best for prediction.	The student found a model, indicated the domain for the model, and discussed why it was the best model.	The student found a model, gave its domain, discussed why it was the best model, and supported why or why not the model should be used for prediction of sales in 2015.
6	The student did not determine if Q1 sales were a predictor for Total Annual Sales	The student attempted to determine if Q1 sales were a predictor for Total Annual Sales. His conclusion was not correct or he had a correct conclusion with no support	The student correctly determined if Q1 sales were a predictor for Total Annual Sales. He attempted to support this answer using mathematics, but the support was not complete.	The student correctly determined if Q1 sales were a predictor for Total Annual Sales and accurately supported this answer using mathematics.
Neatness	The document was not neat or well organized	The document showed neatness and organization in at least one section.	The document was mostly neat and well organized.	The document was neat and well organized.

Assessment List for Shuffle or Nano?

Number	Element	Point Value	Self	Teacher
1	Student made three or more accurate statements about the data in the chart.	4		
2	Student made a graph. He labeled the axis correctly and had the scales correct.	4		
3	The student explored at least two models, determined one was more appropriate, correctly discussed why his choice was the most appropriate model, and discussed its accuracy.	4		
4	The student identified at least two trends in various sections of the graphs and correctly suggested a model that would fit each of these trends.	4		
5	The student found a model, gave its domain, discussed why it was the best model, and supported why or why not the model should be used for prediction of sales in 2015.	4		
6	The student correctly determined if Q1 sales were a predictor for Total Annual Sales and accurately supported this answer using mathematics.	4		
7	The document was neat and well organized.	4		
	Total	28		